Calibrating accelerometers

Modern electronic devices are smart: they can be used as a navigator or pedometer, they can automatically rotate the screen and recognize whether the user is walking or running, riding a bus or a cab. All these and many other tasks are performed using tools called *accelerometers*. The simplest (single-channel) accelerometer has a specific direction, its sensitivity axis; the readings of an immobile accelerometer allow us to calculate the deviation of its axis from the vertical axis directed downward, i.e. from the direction of the gravity force. If several single-channel accelerometers are attached to a device, then its spatial orientation as a whole can be determined from the directions of the accelerometers' axes.

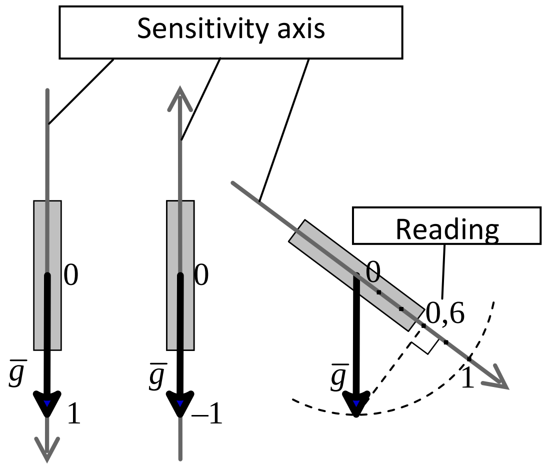
However, miniature accelerometers are unavoidably manufactured with some defects that lead to errors in their readings. Defects in sensors produce a systematic shift of all readings by a certain amount and a multiplicative error which scales all readings by a certain factor. Another kind of errors caused both by the manufacturing process and by inaccurate mounting of sensors in the accelerometer's case can lead to a slight deviation of the sensitivity axis from the axis of the case.

To find out exactly how a particular accelerometer distorts its readings and perform a digital correction of these distortions, a calibration procedure is carried out. One method of calibration is to take the accelerometer readings in several precisely fixed positions of its case and, using the data thus obtained, to create a formula relating the distorted accelerometer readings to its position. This formula can then be used to determine the orientation of the accelerometer in an arbitrary position.

Your task is to create formulas that allow one to calculate the expected readings of real (distorted) accelerometers given the position of their case in space. To find the formulas, use the provided calibration data sets. This task must be completed for three levels of increasing difficulty.

Furthermore, for each level, an additional task is proposed, by solving which you can increase the overall assessment of your work provided that the main task has been solved. In this task, it is required to solve the inverse problem, i.e. to create the formulas and/or describe a method that allows one to calculate the spatial position of the case of a set of accelerometers given the real accelerometer readings.

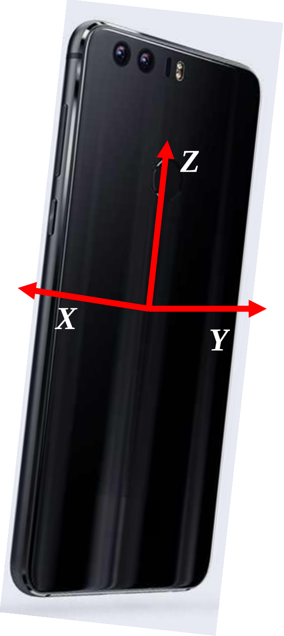
On all levels, the readings of the accelerometers are given as a sequence of values recorded at equal small time intervals in the process of placing their case in a given set of fixed positions and turning it from position to position; unit of measure is the gravitational acceleration *g*. Thus, the reading of an ideal (non-distorted) accelerometer, whose sensitivity axis is directed vertically downwards (i.e., along vector $\bar{g}$) is equal to 1 and if the axis is directed upwards, to –1. If the axis is inclined, then the reading of the ideal accelerometer is equal to the projection of the gravitational acceleration vector $\bar{g}$ onto the axis (see the figure).



Level 1. Single-channel accelerometer

Here, the sensitivity axis is considered to exactly coincide with that of the accelerometer's case. Its readings are recorded in two positions:
a. the axis is directed vertically downwards;
b. the axis is directed vertically upwards.
On Level 1, assume that the position of the accelerometer changes only in one (vertical) plane, i.e. it is completely determined by the angle between the axis of the accelerometer and the vertical direction.
Data: files 1D_f_eng.xls and 1D_g_eng.xls.

Level 2. Dual-channel accelerometer



The accelerometer consists of two single-channel accelerometers X and Z rigidly connected into a single unit in a common case. By design, they must be directed along the X and Z axes of the case, but due to mounting defects they have slight deviations in the XZ plane. Their readings are measured in four case positions described in a separate file by the projections of the vector $\bar{g}$ on its axes (in each position either X or Z axis is directed strictly up or down).

On Level 2, assume that the position of the accelerometer changes only in its XZ plane, while its Y axis remains horizontal and stationary.

Data: files 2D_f_eng.xls and 2D_g_eng.xls.

Level 3. Three-channel accelerometer

The accelerometer consists of three single-channel ones, rigidly connected into a single unit inside a common case. By design, their sensitivity axes are directed along the X, Y, and Z axes of the case, but mounting defects cause deviation of the sensitivity axes from these directions. Accelerometer readings are measured in 20 positions, described in a separate file.

On Level 3, there are no restrictions on the position of the accelerometer.

Data: files 3D_f_eng.xls and 3D_g_eng.xls.

Note 1. If you have solved Level 2 or 3 tasks, then it is not necessary to solve the previous ones. I.e., if Level 2 task is solved, you can skip Level 1, and if Level 3 is solved, you can skip both Levels 1 and 2.

Note 2. In the files *D_g_eng.xls, the rows with the projections of $\bar{g}$ on the case axes have the same order as the case positions change during the measurements.

Hint. To understand how to extract the necessary information from the measurement data, it will be helpful to plot and study the graphs of accelerometer readings as a function of time.