Math Around Us
Senior League. 2018/10/30

1. Two cubes with a digit from 0 to 9 on each of their 12 faces can be used to make a calendar if the digits are distributed in a special way, such that any date in a month from 01 to 31 can be formed from two digits on the front faces of the cubes placed side by side. Santa Claus, preparing New Year gifts, decides to use this idea to make a set of souvenir desk calendars (see the figure). He wants to make it so that each calendar (pair of cubes) in the set is different from any other. What is the greatest possible number of calendars in such a set? More exactly, we say that two cubes are numbered identically if they have the same set of six digits on their faces and they can be put on a desk so that their top faces bear the same digit, front faces bear the same digit, and so on. Position of a digit on a face is of no importance. Two calendars are said to be different if the numbering on at least one cube in one pair is not identical to the numbering of each cube of the other pair.

2. It has been noticed long ago that a year of life for a little kid lasts for an eternity, while for a senior person it flies by almost instantly. One explanation is that after a person has lived a time t, it seems to her that she has lived a certain time $x(t)$ (distorted in her mind with respect to t). Moreover, the next small interval Δt of real time is perceived by her as a “personal” interval Δx, also distorted, which seems to her $x(t)$ times shorter than the real one. Then the greater is the magnitude of $x(t)$, the smaller interval Δx seems to be for the same Δt. Try to find the function $x(t)$, the “interior age” of a person.

3. About 70 years ago the Swedish company TetraPak introduced original milk packs of tetrahedral shape. The main advantage of this packing was in that it’s very easy to manufacture: a strip of special cardboard is rolled into a tube, which is filled with milk and then successively squeezed by clamps of alternating perpendicular directions (Fig. 1) and welded along the edges thus formed. Then a pack is cut off from the bottom of this chain of pyramids, the chain moves one position down and the process is repeated. The packs were transported in special containers (Fig. 2, 3) with bottom in the shape of a hexagonal pyramid, on whose faces the first layer of packs was placed and in the gaps between them, the packs of the second layer were inserted. The top of the second layer formed a hollow with six triangular faces on which the third layer of six packs was laid. Find the width of the cardboard strip needed to obtain packs of volume $V = 0{,}5$ l.

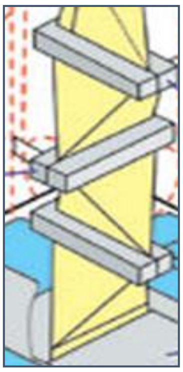

Fig. 1

Fig. 2

Fig. 3