Calibrating accelerometers

Modern smart electronic devices can perform as a navigator, pedometer, recognize how the user moves – walks, runs, rides in a taxi or on a bus, and are capable to automatically orient the image on the screen. In solving all these and many other tasks, so-called *accelerometers* are used. The simplest (single-channel) accelerometer has a specific direction – its sensitivity axis; the readings of the motionless accelerometer allow to calculate the deviation of its axis from the downward vertical, i.e. from the direction of gravity. If several single-channel accelerometers are attached to the device, then the orientation in space of the device as a whole can be determined from the positions of the accelerometers' axes.

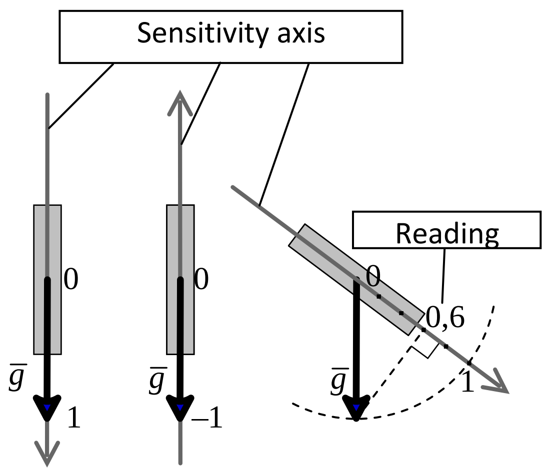
However, in the manufacture of miniature accelerometers it is impossible to completely avoid defects. Defects in sensors lead to such errors in readings as the systematic shift of all readings by a certain amount and their proportional change, i.e. increase or decrease a certain number of times. Defects in mounting or manufacture of the sensors in the accelerometer case can lead to a slight deviation of their sensitivity axes from the axes of the case.

To find out exactly how a particular accelerometer distorts its readings, and make a digital correction of these distortions, the calibration procedure is carried out. One method of calibration is to take the accelerometer readings in several precisely fixed positions of its case and, using the data, create a formula linking the accelerometer readings, subjected to distortion, with its position. This formula can then be used to determine the orientation of the accelerometer in an arbitrary position.

Your task is to build formulas that allow to calculate the expected readings of real (distorted) accelerometers, if the position in the space of their case is given. Use the provided calibration data sets to build the formulas. This task should be completed for three tiers of increasing difficulty.

Furthermore, on each tier, an additional task is proposed, the solving of which can increase the overall assessment of your work, provided that the main task has been solved. In this task, it is required to solve the inverse problem, i.e. to build the formulas and/or describe a method that allows calculating the spatial position of the common accelerometers' case on the basis of the real accelerometer readings.

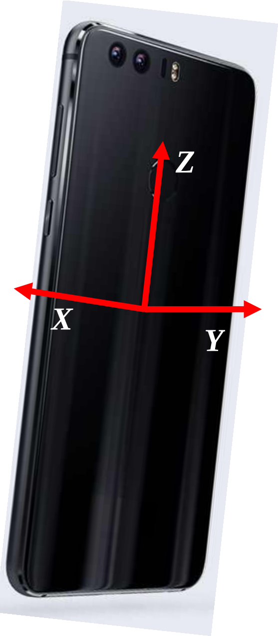
On all tiers, the readings of the accelerometers are given as a sequence of values recorded at equal small time-intervals in the process of installing their case at given motionless positions and turning it from position to position; unit of measure is the gravitational acceleration g. Thus, the reading of an ideal (non-distorted) accelerometer, the sensitivity axis of which is directed vertically downwards (i.e., along the vector $\bar{g}$), will be equal to 1, and if the axis is directed upwards then –1. If the axis is directed at an angle to the vertical then the reading of the ideal accelerometer is equal to the projection of the gravitational acceleration vector $\bar{g}$ onto the axis (see figure).



Tier 1. Single-channel accelerometer

Here, the sensitivity axis is considered to be exactly the same as the axis of the accelerometer case. Its readings are recorded in two positions:
a. the axis is directed vertically downwards;
b. the axis is directed vertically upwards.
In Tier 1, assume that the position of the accelerometer changes only in one (vertical) plane, i.e. it is completely determined by the angle between the axis of the accelerometer and the vertical.
Data: files 1D_f_eng.xls and 1D_g_eng.xls.

Tier 2. Dual-channel accelerometer



The accelerometer consists of two single-channel accelerometers X and Z, which are rigidly connected into a single unit in a common case, structurally directed along the X and Z axes of the case, but due to mounting defects having slight deviations in the XZ plane. Their readings are measured in four case positions described in a separate file by the projections of the vector $\bar{g}$ on its axes (in each position one of the X and Z axes is directed straight up or down).

In Tier 2, assume that the position of the accelerometer changes only in its XZ plane, while its Y axis remains horizontal and stationary.

Data: files 2D_f_eng.xls and 2D_g_eng.xls.

Tier 3. Three-channel accelerometer

The accelerometer consists of three single-channel ones, rigidly connected into a single unit in a common case. Their sensitivity axes are structurally directed along the X, Y, and Z axes of the body, but mounting defects lead to deviations of the sensitivity axes. Accelerometer readings are measured in 20 positions, described in a separate file.

In Tier 3, there are no restrictions on the position of the accelerometer.

Data: files 3D_f_eng.xls and 3D_g_eng.xls.

Note 1. If you have solved the tier's 2 or 3 task, then solving the previous ones is not necessary. I.e., if tier 2 is solved, then tier 1 is not necessary, and if tier 3 is solved, then tiers 1 and 2 are not necessary.

Note 2. In the files *D_g_eng.xls, the lines with the projections of $\bar{g}$ on the axes of the case go in the same order as the case positions change during the measurements.

Hint. To understand how to extract the necessary information from the measurement data, build and study the graphs of accelerometers readings as the function of time.