Combinatorics and Logic

Senior League

1. How to rearrange the numbers $1, 2, \ldots, 2017$ into a sequence $a_1, a_2, \ldots, a_{2017}$ such that expression

$$1^2 \cdot a_1 + 2^2 \cdot a_2 + \ldots + 2017^2 \cdot a_{2017}$$

would hold maximum value? *(I. N. Sergeev)*

2. In a dark storage room 24 slippers are scattered that originally formed 12 pairs of 3 different colours and 4 different styles (all pairs were different from each other). What is the least number of slippers that salesman should take from the room to be sure that he can show a customer three pairs of slippers of three different styles and three different colours? *(S. B. Gashkov)*

3. In the set of positive integers a subset is selected. For every positive integer n define $n?$ as follows: in expression $n! = 1 \cdot 2 \cdot \ldots \cdot n$ move from numerator to denominator all numbers that belong to the subset. We want to select the subset in such way that all sufficiently large positive even n would satisfy inequality

$$|n? - 1| < 10^{-2017}.$$

Is it possible? *(I. N. Sergeev)*

4. Six different gift sets are arranged from souvenirs of 7 kinds, each set contains 3 different souvenirs. Is it possible to say for sure that some two of these gift sets contain *exactly* 1 common souvenir? *(S. B. Gashkov)*

5. 40 red and 50 green stones lie on a table. Two players, Petya and Vasya, take turns: each turn a player picks a color and removes an arbitrary (at his choice) number of stones of this color that is a divisor of the number of stones of the other color (zero is divisible by every positive integer). Whoever takes the last stone wins. Who has a guaranteed win, given that Petya is first? *(suggested by I. A. Sheipak)*