

Algebra and Number Theory

Senior League

1. Solve the equation

$$3p^q + 3q^p = n!,$$

where p, q are prime numbers and n is a positive integer.

2. Let a, b, c be the lengths of the sides of some triangle with perimeter of no more than 2π . Prove that $\sin a, \sin b, \sin c$ are also lengths of the sides of some triangle.
3. Let us call a positive integer n *funny* if for every its positive divisor d number $d+2$ is prime.
 - (a) Determine the largest number of divisors that a funny number can have.
 - (b) Find all funny numbers with the largest number of divisors.
4. Given positive numbers a, b, c such that $ab + bc + ac = 1$, prove

$$\sqrt[3]{\frac{1}{a} + 6b} + \sqrt[3]{\frac{1}{b} + 6c} + \sqrt[3]{\frac{1}{c} + 6a} \leq \frac{1}{abc}.$$