The 9th Mathematical Multiathlon

31 October–5 November 2016, Moscow

Senior League
Team Contest

1. In a triangle ABC with angle $\alpha = 15°$ the sides denoted by *a,b,c* and satisfy the equation $b = \sqrt{a(a+c)}$. Find the area of ABC if the radius of inscribed circle $r = 1$. Write the answer as a sum of numbers.

2. To model a process a physicist has obtained the function $y_1(t) = 2^{-4t}$. An engeneer for the same process has obtained the function $y_2(t)$, that turns out to be inverse one for $y_1(t)$. Scientific dispute has been stopped by the mathematician. He proves that both functions are close to hyperbolic one $y_3(t) = \frac{c}{t}$ in the range of $t_1 \le t \le t_2$. The *c* is a constant, and for equations $y_1(t) = y_3(t)$ and $y_2(t) = y_3(t)$ there are two solutions in the given range, that can easily be guessed if *c* is known.

1) Find *c* and minimal interval $[t_1, t_2]$ for the statement of mathematician.
2) Find out how many solutions of $y_1(t) = y_2(t)$.

3. A tin sheet with area $2a^2$ is used to make a closed box with the shape of a cuboid with maximum volume $V_m = maxV$.

1) Find the sides values of such box. The sheet can be deformated only with straight lines and cutted. But it is impossible to connect (glue or welding) the different parts of the sheet.
2) The same question about $maxV$ and the sides values of opened box (without lid) that can be made from a tin sheet with sizes $b \cdot c$. Find $maxV$ if the perimeter is known.

4. Find the parameter *a* (in centimeters) such that locus of points in the coordinate plane *(x, y)*, satisfying the equation
$(|x - 2a| + |x + 2a| + |2y - a| + |2y + a|) \cdot cm = a^2 - 7cm^2$,contains the rhomb with maximum area?
Can the area *S* of the rhomb be more than $100cm^2$? If it can, find the diagonals length of this rhomb; if not – prove it.

5. Find any polynomial of third degree $P_3(x) = ax^3 + bx^2 + cx + d$, which in the range $0 \le x < \frac{1}{2}$ satisfies the inequality

$$\left| P_3(x) - \frac{x}{1 - \frac{x}{2}} \right| < \frac{1}{75}$$

6. A radar detected unidentified flying object (UFO), which is frozen at the map and forms the set of points $25x^2 + xy + y^2 + 16x + 2y + 3 \le 0$. Find the extreme azimuths values of the UFO, i.e. the angle with vertex at the origin (the location of the radar) that contains the UFO. Rate the *minR* – the minimum distance of the UFO from radar station. Can *R* be less than 0.5?

7. Three numbers *x, y, z* and *a, b, c* satisfy the system of equations

$$\begin{cases} 12x^2 + 4y^2 + 9z^2 = 20 \\ 9a^2 + b^2 + 3c^2 = 25 \end{cases}$$

What is the range of the expression *2cx-2ay+bz*?