

The 9th Mathematical Multiathlon

31 October–5 November 2016, Moscow

Geometry

Senior League

1. A circle inscribed in the right angle of an isosceles right triangle divides the hypotenuse into three equal parts. The leg of the triangle is of length 1. Find the radius of the circle.
2. The length of each bisectors of a triangle is at least one unit. Prove that its area is at least $\sqrt{3}/3$.
3. A chain of three equilateral triangles ABC , CDE , and EFG , whose vertices are written counterclockwise, is arranged so that D is the midpoint of AG . Prove that triangle BFD is also equilateral.
4. For any four vertices of a convex polyhedron, the tetrahedron with these vertices lies inside the polyhedron. Does there exist a polyhedron such that none of such tetrahedrons lies inside it?