

Combinatorics and Logic

Senior League

1. Let S be the set of all partitions of 2000 in a sum of positive integers. For every such partition p , we define $f(p)$ to be the sum of *the number of summands in p* and *the maximal summand in p* . Compute the minimum of $f(p)$ when $p \in S$.
2. Is it possible to place several pieces on a checkered 2016×2016 board so that on every diagonal (including one-cell diagonals) there was an odd number of pieces?
3. 100 fatties stand in a row. Every two consecutive fatties differ no more than 1 kg in weight. Prove that all of them can be distributed in a motorcade of 25 identical cars (4 fatties in each car) so that every two consecutive "loaded" cars would also differ no more than 1 kg in weight.
4. In two cells of an $n \times n$ table there are signs "−", and in all other cells there are "+". Each turn it is allowed to reverse all signs in one column or in one row of the table. After several turns there were exactly 9 signs "−" in the table. What is the maximum value of n for which it is possible?
5. Some cities in a country are connected by two-way flights, and the number of flights is exactly 5 times greater than the number of cities. Prove that the number of two-way routes with exactly one change is at least 45 times greater than the number of cities. (*Two-way route* means that routes $A-B-C$ and $C-B-A$ count as one. Route endpoints are always distinct.)