VIII tournament "Mathematical all-round"
2–7 November 2015, Moscow

Combinatorics

Senior League

1. Is it possible to arrange on a circle natural numbers $1, \dots, 2016$, so that all 2016 sums of two neighbouring numbers are distinct?

2. Let $n > 5$. We consider edge colourings of a complete graph K_n with red and green such that red graph and green graph are connected. Prove that there exists at least $(n-1)!(n+1)$ such colourings.

3. Each positive integer shall be coloured red or green such that it satisfies the following properties:

   (a) if a and b (not necesserily distinct) are of the same colour, then $3a + b$ is of that colour, too;

   (b) There are red and green numbers.

   Find all such colorations.

4. Talented boy Petya Tort thinks of a natural number from 1 to 1000. You can ask him questions of the form «Does your number belong to X?». Answering the first question he says truth, but at some moment he begins to lie and lie until the end of conversation (this can happen even after the last question). What is the least number of questions that must be asked in order to know what the number is for sure?

5. Prove that for every natural n there exist n non-intersecting hamiltonian cycles in a complete graph K_{2n+1}. (non-self-intersecting cycle is said to be hamiltonian if its lenght is equal to $2n + 1$)