

## The 8th Mathematical Multiathlon

2–7 November 2015, Moscow

### Algebra and number theory

#### Senior League

1. Two positive integers are said to be *neighbors* if their decimal notations differ in only one decimal place (e.g., the numbers 23578 and 23478 are neighboring). What is the greatest possible number of  $n$ -digital integers no two of which are neighbors?

2. Solve the equation

$$x + \frac{x}{\sqrt{x^2 - 1}} = \frac{35}{12}.$$

3. Find all positive integers  $a$ ,  $b$ , and  $c$  satisfying the condition

$$a^b + b^c = abc.$$

4. A polynomial with integer coefficients is said to be *economical* if its leading coefficient is 1 and the set of all its other coefficients, including zero coefficients, coincides with the set of its roots taken with their multiplicities, that is, if exactly  $m$  of the coefficients are equal to  $a$ , then  $a$  is the polynomial's root of multiplicity  $m$ . Find all economical polynomials of degree  $n$  for (a) (1 point)  $n = 2$ , (b) (2 points)  $n = 3$ , (c) (4 points)  $n = 4$ . (A number  $x_0$  is a root of multiplicity  $m$  of a polynomial  $P(x)$  if  $P(x) = (x - x_0)^m Q(x)$ , where  $Q(x_0) \neq 0$ .)