

Problem 1

It often happens that as someone is telling a story to a company of people, **all of a sudden everyone in the group start laughing at the same time**. We would like to understand the **logical** essence of what is funny. Which of the following characteristics of a story's ending make the audience laugh—perhaps not just one feature, but their combination?

The ending can be described:

1. as **absolutely** logical;
2. as logical in a **peculiar** way, perhaps even with some inconsistencies;
3. as absolutely **illogical** or absurd, perhaps even just stupid;
4. as completely and **unambiguously** predictable;
5. as **ambiguously** predictable, perhaps even in many ways;
6. as absolutely **unpredictable** or unexpected;
7. in a way **different** from those listed above in terms of logic. If so, exactly how would you describe it?

Try to justify your answer, the best justification being an example of a specific funny story (anecdote) that makes readers or listeners laugh precisely because it has the features you have chosen from the list above.

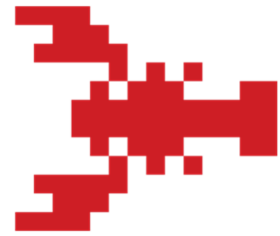
Answer: Features 2 and 6.

Solution

Let's give an example that confirms that a good joke results from absolute unpredictability or unexpectedness (6) in combination with peculiar logic, perhaps even with some inconsistencies (2).

Someone regularly visited a bar and always ordered exactly three shots of liquor. Once the bartender asked him: "Why do you always order three shots?" The customer answered: "I have two friends and we agreed that whenever one of us drinks, he drinks for all the three at once." But one day the customer comes to the bar and orders only two shots. The bartender takes out a handkerchief, dabs at a tear, and asks: "What, did one of your friends die?" "No," the man replies, "I just quit drinking."

In this example, the bartender gave a predictable and logical explanation for what was happening, namely: a friend died. But the customer offered a completely unexpected version—not entirely logical, yet still justified by a peculiar logic: on the one hand, if he quit drinking, he shouldn't drink at all, but on the other hand, *he* did quit drinking, but *his friends* didn't.



Problem 2

In this problem we assume that the Earth is a sphere of radius 6400 km.

1. A rope was tightly stretched around the equator. Then it was decided to make it longer. What is the **smallest integer** number of meters by which its length must be increased so that:

- a) the distance between any point of the rope and the equator is at least 1 m;
- b) it would be possible to pull the rope up from the Earth at one point so that a pillar of height 150 m located on the equator could fit under the rope?

You can use the following approximation: for values of α (in radians) close to zero, $\tan \alpha \cong \alpha + \alpha^3/3$.

2. Two lighthouses are 50 km apart (measured along the sea surface), and their lamps are at the same height h m above sea level. What is the **smallest integer** h for which one lamp can be seen from the other one?

Answer: 1.a) 7; 1.b) 2; 2. 49.

Solution

1.a) The elongation that provides the required clearance is equal to

$$\Delta l = 2\pi(R + 1) - 2\pi R = 2\pi = 6.28... \text{ (m)},$$

where R is the Earth's radius. Notice that Δl does not depend on the radius.

1.b) Suppose that the rope is lengthened by 1 m and pulled at its point A (so that A is its highest point). Let B be a point of contact of the rope and the equator closest to A (see the figure). Then the angle $\alpha = \angle AOB$, where O is the Earth's center, satisfies the equation

$$\frac{1}{2} = R \tan \alpha - R\alpha \cong \frac{R\alpha^3}{3}, \text{ or, for } R = 6400000,$$

$\alpha = \sqrt[3]{3(\tan \alpha - \alpha)} \cong \sqrt[3]{\frac{3}{2R}} = 0,0061...$, and so the height of point A is

$$h = R \left(\frac{1}{\cos \alpha} - 1 \right) \cong 121,6... < 150 \text{ (m)}.$$

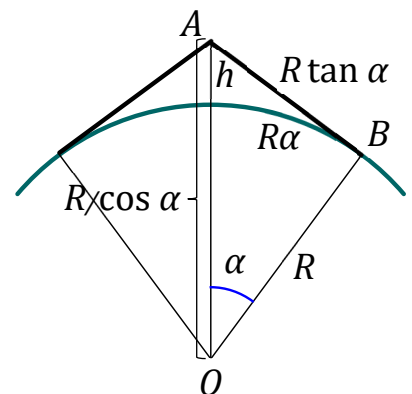
Similarly, if the rope is lengthened by 2 m, we get $h \cong 193,1... > 150$ (m). Therefore, the smallest possible integer elongation is 2 m.

The same answer can be obtained by computing Δl for $h = 150$ m and justifying that an increase in elongation Δl entails an increase in height h :

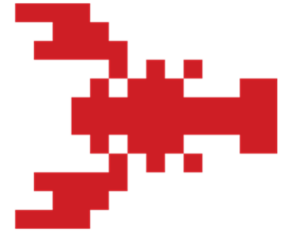
$$\alpha = \arccos \frac{R}{R+h} = 0,0068..., \quad \Delta l = 2R(\tan \alpha - \alpha) = 1,34...$$

2. We can use the same figure, in which A is now one of the lighthouses, AB is the direction from this lighthouse to the other one (which is not shown), and the angle between the radii drawn to the lighthouses is 2α . Then

$$\alpha = \frac{50/2}{R} = 0,0039... \text{ and } h \geq R \left(\frac{1}{\cos \alpha} - 1 \right) \cong 48,8... \text{ (m)}.$$



Problem 3



A train moves along a straight road with constant speed. A passenger at the window observes the trees running past outside. At some moment, two trees occur on the line of sight, one twice as far as the other. How many times is the angular velocity of the nearer tree greater than that of the farther tree if the line of sight:

a) is perpendicular to the railway tracks;

b) forms a given angle α with the perpendicular to the tracks?

Assuming that at time $t = 0$ the direction to a certain tree is perpendicular to the tracks (i.e., $\alpha = 0$), and the angular velocity ω of this tree is equal to 1, express ω in terms of:

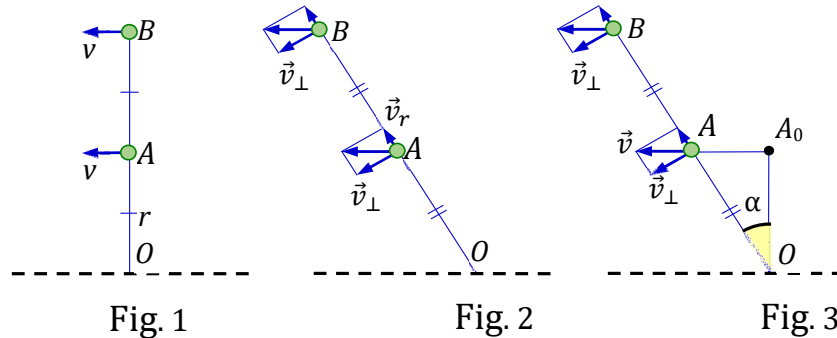
c) the angle α ;

d) time t .

Answer: a) 2; b) 2; c) $\cos^2 \alpha$; d) $\frac{1}{1+t^2}$.

Solution

a) Denote the velocities of both trees A and B with respect to the observer O by $\vec{v}$ (Fig. 1; we assume that the velocity of the train with respect to Earth equals $-\vec{v}$). Then, putting $|\vec{v}| = v$ and using a well-known formula from physics, for the angular velocities of trees B and A we get the equations $\omega_B = \frac{v}{OB}$ and $\omega_A = \frac{v}{OA} = \frac{v}{OB/2} = 2\omega_B$.



b) If $\alpha \neq 0$, we decompose the linear velocity $\vec{v}$ of the trees into two orthogonal vectors, one of which is directed along the tree's position vector $\vec{r}$: $\vec{v} = \vec{v}_r + \vec{v}_\perp$ (Fig. 2). Then the angular velocity of the tree is $\omega = \frac{v_\perp}{r}$, where $r = |\vec{r}|$, and so $\frac{\omega_A}{\omega_B} = \frac{OB}{OA} = 2$.

c) If A_0 and A are the respective positions of the tree for $t = 0$ and for $\angle A_0OA = \alpha$ (Fig. 3), then $OA = \frac{OA_0}{\cos \alpha}$ and $v_\perp = v \cos \alpha$, and by the assumption, $\omega_{A_0} = \frac{v}{OA_0} = 1$. So

$$\omega_A = \frac{v_\perp}{OA} = \frac{v \cos^2 \alpha}{OA_0} = \cos^2 \alpha. \quad (*)$$

d) We have $\omega_A = \cos^2 \alpha = \frac{1}{1+\tan^2 \alpha}$ by (*); but $\tan \alpha = \frac{A_0A}{OA_0} = \frac{vt}{OA_0} = \omega_{A_0} t = t$.