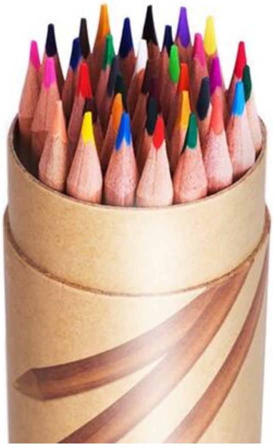
Math Applied to Science Olympiad (MATS/Primate)

Senior League.

Problem 3

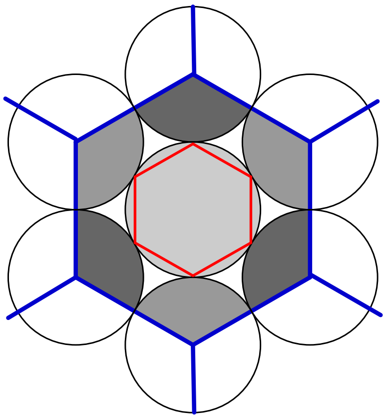


Round wooden pencils arrive at a factory in cylindrical containers of height equal to the pencil's length. Each container holds 3,000 round pencils. At the factory, round pencils are turned into hexagonal (whose cross-section is a regular hexagon) by removing the smallest possible amount of wood.

Estimate the maximum number of hexagonal pencils that the container used for round pencils can hold. You can neglect the boundary effects in your estimation and assume that pencils form the densest possible packing in the container.

Estimate the error caused by neglecting the boundary effects.

Solution:



In the cross-section of the package, one observes a set of equals circles or hexagons. Consider this sets. For the densest packing of circles, there are 6 circles around a given one. The centers of the outer circles form a regular hexagon with the side of $2R$, where R is the circle radius. The circles occupy the area of a whole circle and 6 sectors of the angle of 120°, the latter has the area equal to the area of **three** circles.

The cross-section of the hexagonal pencil is a regular hexagon with the side of R. Its area is ¼ of the area of the above-described hexagon. Since the hexagons cover the plane without spaces for each large hexagon, we have **four** hexagonal pencils

Neglecting the boundary effects (some hexagons and circles fits the package partially), we conclude that for the area occupied by three round pencils is enough to room four hexagonal ones. Thus, the number of hexagonal pencils is 4/3 times large than the number of the round ones.

The boundary effects are significant for the external layer of the pencils only. The package fits approx. 4000 pencils, the area of each is $3\sqrt{3}R^2/2$, thus the area of the package is $6000\sqrt{3}R^2$, and its radius is $(6000\sqrt{3}/\pi)^{1/2}R \approx 55R$. The area of a strip with the width of $2R$, close to the boundary is approximately $2\pi \cdot 55R \cdot 2R = 220\,\pi R^2$. It means that the boundary effects are significant for approximately 220 pencils. Assume that it is possible to pack a half of them, so the error of the quantity is approx. 100 pencils, and the relative error is 110/3000 for the round pencils and 110/4000 for hexagonals. The overall relative error is 11/3000 + 11/4000 ≈ 6%

Answer: 4000, 6%.