Math Applied to Science Olympiad
(MATS/Primate)
Senior League.
Problem 2

A train goes downhill along a lengthy inclined stretch of track with a slope of 2% (2 meters descend for 100 meters length). There is a speed limit of 36 km/h on this stretch. The driver can put on the brakes to get the deceleration of 0.15 m/s^2. A pause of 10 s must be made between two successive brakings to refill the pneumatic system and let the brakes cool down. What is the maximum possible average speed of the train under these conditions?

Solution:

There are two driving modes for the train: brakes off, acceleration is $a_1 = g \sin\alpha = 0.2$ m/s^2 (тормоз выключен) or brakes on, acceleration is a_2 =-0.15 m/s^2. To make the average speed maximal, the driver should implement the following periodic regime:

1. When the speed limit of V_0 =10 m/s is reached, activate the brakes
2. Deactivate the brakes when the speed falls to the value V_1 such that the speed limit will be reached by the end of the next stage
3. Drive in the accelerating mode with deactivated brakes for $\tau = 10\ s$ until the speed limit is reached.

If the driver increases the duration of stage 3, the value of V_1 should be decreased. Thus, the average speed becomes lower than in the described regime.

The value of V_1 satisfies $V_0 = V_1 + a_1\tau$, $V_1 = V_0 - a_1\tau$.

The average speed in uniformly accelerated motion equals half-sum of maximal and minimal speeds. This means that the average speeds during accelerations and decelerations are the same and equal to

$$V_c = \frac{V_0 + V_1}{2} = \frac{2V_0 - a_1\tau}{2} = 9\ m/s$$

It is the average speed for the whole descend.