

**Math Applied to Science Olympiad
(MATS/Primate)
Senior League.
Problem 1**



Winnie-the-Pooh arrived at Eeyore donkey's house for a party too early, when nobody was at home, and found a whole jar of raspberry jam prepared for the guests. Winnie had a very sweet tooth, so he immediately ate one third of the jar's volume. When it occurred to him that Eeyore would be upset, he refilled the jar with sugar syrup and stirred the mixture. Half an hour later, as nobody showed up and he got hungry, Winnie came back to the jar, ate one third of the mixture again, refilled the jar with syrup, and stirred. Still alone and starving, he kept doing so every half an hour again and again.

How much time, counting from the moment he arrived, did it take for Winnie to consume 90% of the jam?

What was the volume of the syrup he had eaten with the jam by this moment, if the jar's volume is 1 liter?

Assume that Winnie swallows each portion in no time, at one gulp.

Solution:

The concentration of the jam in the jar decreased in geometrical progression:

$$C_{n+1} = \frac{2}{3} C_n$$

The initial concentration is $C_0 = 1$, at the instant $t = 0.5 n$ hours Winnie ate $M_n = \frac{1}{3} C_n$ of jam. Hence we are to find n such that

$$M = M_0 + \dots + M_n \geq 0.9$$

The sum of geometrical progression is

$$M = \frac{1}{3} \frac{1 - \left(\frac{2}{3}\right)^{n+1}}{1 - \frac{2}{3}} = 1 - \left(\frac{2}{3}\right)^{n+1}$$

For n we have

$$\left(\frac{2}{3}\right)^{n+1} < 0.1, \quad n > 5$$

So, Winnie ate 90% of jam in 2.5 hours after he came to the house.

Winnie took a portion of $\frac{1}{3}$ liter 6 times, in total he ate 2 liters of different mixtures, including $(1 - (\frac{2}{3})^6) = \frac{665}{729}$ l of pure jam. Hence volume of the syrup is

$$\frac{6}{3} - \frac{665}{729} = \frac{793}{729} \approx 1.09 \text{ l}$$

Answer: 2,5 hours, 1.09 l