Math Applied to Science Olympiad (MATS/Primate) Junior League. Problem 1

Winnie-the-Pooh arrived at Eeyore donkey's house for a party too early, when nobody was at home, and found a whole jar of raspberry jam prepared for the guests. Winnie had a very sweet tooth, so he immediately ate one third of the jar's volume. When it occurred to him that Eeyore would be upset, he refilled the jar with sugar syrup and stirred the mixture. Time went by, nobody showed up and Winnie got hungry. So he came back to the jar, ate two thirds of the volume, refilled the jar with syrup again and stirred. Some time later he did this again; this time he ate a half of the mixture and refilled the jar with syrup. When Eeyore came home, he realized that something was wrong. Eeyore measured the density of the mixture in the jar: it was 1270 kg/m³. What was the initial density of the jam if the volume of the jar is 1 liter and the density of the syrup is 1300 kg/m³? How much syrup had Winnie-the-Pooh consumed while waiting?

*Решение:

The expression for density is

$$\rho = \frac{\frac{1}{2}\left(\frac{1}{3}\left(\frac{2}{3}V_0\rho_0 + \frac{1}{3}V_0\rho_1\right) + \frac{2}{3}V_0\rho_1\right) + \frac{1}{2}V_0\rho_1}{V_0}$$
$$\rho_0 = 9\rho - 8\rho_1 = 1030 \, kg/m^3$$

To evaluate the mass of the consumed syrup, we find the total mass of added syrup and the mass of the remain:

$$m_0 = \rho_1 V_0\left(\frac{1}{3} + \frac{2}{3} + \frac{1}{2}\right)$$
$$m_{rem} = \frac{8}{9}\rho_1 V_0$$

$$m = m_0 - m_{ost} = 1,95 - 1,3 = 0,65 \, kg = 650 \, g$$

Answer: 1030 kg/m³ , 650 g